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SINGULAR PERTURBATION METHODS APPLIED TO THE DYNAMICS AND RHEOL--ETC(U)
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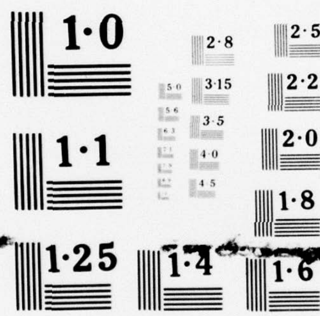
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Department of AERONAUTICS and ASTRONAUTICS
STANFORD UNIVERSITY

Singular Perturbation Methods Applied to the Dynamics and Rheology of Suspensions

Final Report

By I-Dee Chang

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1. Statement of the problem studied.

Many fundamental problems pertaining to the dynamics of suspended particles remain unsolved. Of particular interest to our present investigation are problems belonging to the following four categories:

a. Where the non-linear effect of the fluid is a decisive factor in determing the motion of the suspended particles. An example of such problem is the motion of a particle in a slow shear flow. It was found that a sphere, when moving relative to a fluid under uniform simple shear, experiences a lift force transverse to the direction of fluid and particle motion. If the fluid inertia effect is neglected, as in the case of Stokes approximation, the lift force will disappear and only streamwise drag remains. Thus the lift force, which may cause the sphere to drift across streamlines, is a direct consequence of inertia effects. Harper and Chang have shown both analytically and experimentally for a dumb-bell shaped particle that the resulting lift tends to orient the particle to the orientation of maximum dissipation.

b. Where the inertia of the suspended particle itself is of importance. An example of this is the rotary Browian motion of particles in a suspension.

c. Where the hydrodynamic couplings between the suspended particles and between the particles and the container wall, are important.

d. Where the collective motion of an assembly of particles is of interest. During the present program, methods have been developed to study these types of

problems analytically. In particular, we have concentrated our effort on three representative problems. In these three problems, small but significant physical perturbation effects characteristics in the four types of problem list in § 1 are present. The mathematical techniques required to deal with them are therefore of general interest:

(i) The motion of a small spherical particle suspended in an oscillating shear flow.

(ii) The motion of ellipsoidal particles in a viscous shear flow, and

(iii) unsteady motion of a small particle in a compressible viscous flow.

The above investigations resulted in large amounts of analytical results and numerical data. Some part of these investigations have been published as technical reports and released. Much work is still in progress and will be published as soon as the work is complete.

2. Summary of most important results.

a. The motion of a spherical particle suspended in an oscillating shear flow. The force on a small sphere immersed in an oscillating shear flow is investigated. The unsteady Navier-Stokes equation is solved by the method of matched asympotic expansions. The inertia effects of the fluid produce a lift force on the sphere in the direction of increasing flow velocity. For a neutrally buoyant sphere freely suspended in the flow, this lift force will cause the sphere to drift sidewise across the streamline. The motion of the sphere in a sinusoial oscillating shear flow is examined in detail. The small streamwise slip velocity

between the sphere and the fluid is calculated first. From this, the lift force on the sphere is determined. It is found that the lift force oscillates at twice the frequency of the sheer flow field and has a d-c component which tend to make the sphere drifting slowly across the streamline toward the flow region of faster velocity. The drift velocity of a sphere of 200 μ in a shear flow oscillating at 1 Hz is estimated. The magnitude of this velocity is found to be quite reasonable.

b. <u>The motion of ellipsoidal particles in a viscous shear flow.</u> This investigation is related to the intrinsic viscosity of a dilute suspension of particles in a homogeneous fluid. Jeffery in 1922 calculated the intrinsic viscosity for a dilute suspension of ellipsoidal particles in a linear shear flow using Stokes equations. However, his solution was indeterminate in the sense that the motion, which is periodic, depends upon the initial conditions of release of the particle. We attempt to carry on Jeffrey's work by including the inertia effect of the fluid and to show that the orbit of rotation changes adiabatically over a long period of time.

The slow gyration of an ellipsoid in a shear flow is studies by a singular perturbation method. The small parameter used in the analysis is the shear Reynolds number R_κ defined by $R_\kappa = \frac{\rho\kappa\ell}{\mu}$ where κ is the shear rate and ℓ the characteristic dimension of the ellipsoid. It is found that as $R_\kappa \to 0$, the Jeffrey solution gives the first order uniformly valid solution of the problem. In the next order

approximation, taking into account the fluid inertia effect, a new set of equations is needed.

In this outer region, which is of distance $0(\ell/\ R_\kappa)$ from the body, the flow field is to first-order equivalent to that induced by a set of gyrating dipoles in a shear flow. To match terms it is necessary to introduce a term $0(R_\kappa^{3/2})$ in the inner expansion. This is of smaller order than the second-order inner solution in the expansion which arises from the nonlinear terms through iteration and having the order $0(R_\kappa)$. The order $0(R_\kappa^{3/2})$ inner solution corresponds physically to that of an ellipsoid rotating freely in an unsteady linear flow field composed of arbitrary strain deformation and rotational motion.

A technical report (SUDAAR No. 492) has been released.

c. Unsteady motion of a small particle in a compressible viscous flow. The motion of a small particle suspended in an imcompressible viscous shear flow has been investigated under stady and unsteady conditions. Recent interest in acoustics calls for extension of this effort to the case when the flow is compressible. We consider therefore the problem of slow vibration of a class of axially symmetric bodies moving parallel to the body axis with the velocity $U = U_o e^{i\omega t}$ in a quiescent compressible viscous fluid. It is expected that results obtained from the anlysis will be useful in many realistic physical situations where in the flow fluctuations are coupled with the motion of the small particle.

There are three non-dimensional parameters in the problem. These are the flow Reynolds number $Re = \frac{\rho_o U_o a}{\mu_o}$, the unsteady Reynolds number $\alpha^2 = \frac{\rho_o \omega a^2}{\mu_o}$

and the Mach number $M = \frac{U_o}{C_o}$, where a is the characteristic dimension of the particle, and quantities with subscript "o" refer to undisturbed flow conditions. Since the particle is small, we assume that Re and α^2 are small. It follows that the Mach number M is also small. We therefore solve the problem by finding the asympotic solution of the exact compressible, viscous heat conductiong Navier-Stokes equations, for small values of Re, α and M . The problem differs from the acoustic-type problem in that viscous effect is important and must be considered. It is found that the solution depends on four characteristic length scales. These are the geometric length scale a , the viscous length scale $\frac{\mu_o}{\rho_o U_o}$, "Stokes" length scale $\sqrt{\frac{\mu_o}{\rho_o \omega}}$ and the wave length λ . The condition $M \ll 1$ in particular implies $\frac{a}{\lambda} \ll 1$. In the inner region $r \ll \lambda$, the flow is nearly incompressible. This greatly reduces the complexity of the problem in the inner region. In the outer region, $r = 0(\lambda)$, the flow must be treated as compressible. However, simplification is derived from the boundary condition and again the problem becomes tractable. In fact, to the order $0(\alpha)$, the flow field is characterized by the "Stokes drag" experienced by the particle rather by the detailed geometry of the particle. The solution of the far field is obtained by means of Fourier transform. It is found that sound waves similar to that generated by dipole dominate the far field. The acoustic intensity pattern shows maxima both in the forward and rearward direction of the motion. The details are however modified by the effects of viscosity and heat conduction.

3. Publications

   a. The motion of a small spherical particle suspended in an oscillating shear flow. By I-Dee Chang, SUDAAR No. 487 August, 1974.

   b. The motion of ellipsoidal particles in a viscous shear flow. By I-Dee Chang SUDAAR No. 492 April 1975

   c. Unsteady motion of a small particle in a compressible viscous flow. By I-Dee Chang (To appear).

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The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Suspensions, viscous flow, non-linear effects, low Reynolds number

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Non-linear hydrodynamic interaction on particles suspended in viscous flows was investigated analytically by the method of matched expansions. In particular, three problems of fundamental importance were analyzed. The first problem involves the motion of a particle in an oscillating viscous shear flow. It was found that because of the inertia of the fluid, a lift force is produced on the particle. This force has a d-c component which tends to cause a sidewise drift of the particle. The second problem involves the extension of Jeffery's classical work on the motion of ellipsoidal particle (continued)

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immersed in a viscous shear flow. It was found that the first order outer solution corresponds to gyrating dipoles in a shear flow. The effect of the shear produces a higher order effect than the one caused by the second order non-linear effect in the near field. The third problem dealt with the motion of a spherical particle in a viscous compressible flow. The flow field was calculated in terms of a near field and a far field. The flow pattern is rather complex because of the combined effects of heat conduction and viscosity.